INVISIBLE PROOF: THE ROLE OF GESTURES AND ACTION IN PROOF

Caroline C. Williams
Univ. of Wisconsin – Madison
ccwilliams3@wisc.edu

Candace Walkington
Univ. of Wisconsin – Madison
cwalkington@wisc.edu

Rachaya Srisurichan
Univ. of Wisconsin – Madison
srisurichan@wisc.edu

Elizabeth Pier
Univ. of Wisconsin – Madison
epier@wisc.edu

Martha Alibali
Univ. of Wisconsin – Madison
mwalibali@wisc.edu

acts and transform the gesturer's cognition (Goldin-Meadow & Beilock, 2010; Alibali & Kita, 2010).

Gestures are a particular form of action that represents the world, rather than acts upon the world directly (Goldin-Meadow & Beilock, 2010). Furthermore, gestures are more than mere movements; as McNeill (1992)

conjecture. In particular, we used *multimodal analysis* techniques (Alibali & Nathan, in press; McNeill, 1992), and coded how each student used action and gesture during the ascertaining and persuading phases of their proof (Harel & Sowder, 1998). For the purposes of this paper, we selected two examples of justification that highlight the role of gesture in mathematical proof, and in our presentation we will provide additional examples.

Results

We share two specific examples: an example of how simulated action (manifested through gesture) can illuminate the ascertaining phase of proof, and an example of embodied proof that demonstrates the role of paired

Simulated Action Illuminates Ascertaining

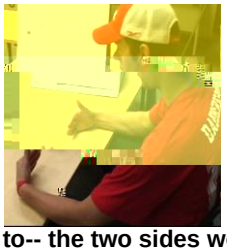
To illustrate the illumination of proof by simulated action, we share an excerpt from the Gears conjecture (Table 1), as the participant leverages her body as a tool for simulating the actions of the gears and identifying parity (shown in Figure 1). The excerpt contains both ascertaining and persuading phases of the proof, and is annotated as such.

For each idea the speaker expresses verbally in the ascertaining phase, she also produces gestures that physically simulated the motions of the gears. By using her body, she is able to understand the relationship of individual gears to each other, and consequently to solve the conjecture. Her eyes remain focused on her hands, as she uses her body to understand *why* the conjecture is true. During the persuading phase, gesture is also critical; as she speaks, her gestural simulations *show* the audience how and why her verbal statement is true. However, a more powerful example of how action supports and constitutes mathematical proof during the persuading phase is given next.

1 **Say one side's 5.**

2 **If the other two sides weren't at least 5 or even equal to 5,**

3 **if you tried to connect them, you think about it, they'd be too short to touch.**



4 **So they would have to-- the two sides would have to be longer**

5 **than the other remaining side.**

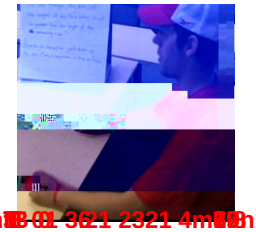


Figure 2: Paired Gesture and Speech Persuading

Paired Gesture and Speech Persuading

To illustrate the pairing of gesture and speech to support persuading in embodied proof, we examine an excerpt that occurs after the participant has solved the Triangle Inequality conjecture (Table 1) and has shifted into the persuading phase of the proof (Figure 2). The verbal element of the proof provides a specific example, as simultaneously the gestural components communicate the generalizability of the participant's proof.

In Figure 2, the verbal and gestural components are woven together to provide a complete

Extending the examination of proof production into gesture and action allows us to conceptualize a more complete model of cognition (Shapiro, 2011), and consequently allows us to design new activities that more coherently account for different strategies of proof production.

Our research provides a startin